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DISCRETE MATHEMATICS FOR COMMUNICATIONS SYSTEMS

ONR CONTRACT NOOO14-84-K-0189

Annual Report

1 February 1984 - 31 January 1985

Solomon W. Golomb

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FIRST ANNUAL REPORT

FEBRUARY 1985

A summary of the work accomplished under ONR Contract No. NOOO14-84-K-0189 is presented for the year February 1, 1984 to January 31, 1985. This is accomplished by presenting the abstracts of the papers published and those that are in progress. In addition, copies of papers published are attached.

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-S.W. Golomb, "Algebraic Constructions for Costas Arrays", *Journal of Combinatorial Theory, Series A*, Vol. 37, No. 1, July 1984, pp. 13-21.

ALGEBRAIC CONSTRUCTIONS FOR COSTAS ARRAYS

ABSTRACT

The following is equivalent to a problem posed by John P. Costas [1], who encountered it in the context of attempting to construct sonar signal patterns.

Problem.

For each positive integer n, construct an n×n permutation matrix with the property that the $\binom{n}{2}$ vectors connecting two 1's of the matrix are all distinct as vectors. (That is, no two vectors are equal in both magnitude and slope.)

Thus if $a_{i_1,j_1} = a_{i_2,j_2} = a_{i_3,j_3} = a_{i_4,j_4} = 1$ in the matrix, we must not have $(i_2 - i_1, j_2 - j_1) = (i_4 - i_3, j_4 - j_3)$, nor may we have $(i_2 - i_1, j_2 - j_1) = (i_3 - i_2, j_3 - j_2)$.

Such matrices have been called either Costas Arrays or constellations in Ref. [2], which explores constructions as well as applications for these patterns. It is convenient to represent these arrays on an n×n grid, using dots for the 1's and blanks for the 0's of the matrix. Three examples of 6×6 Costas arrays are shown in Fig. 1.

Previous constructions [2] for Costas Arrays, for special values of n, have been discovered by L.R. Welch and by A. Lempel. This paper contains the first published proofs of the validity of the Welch and Lempel constructions, as well as a major new construction.

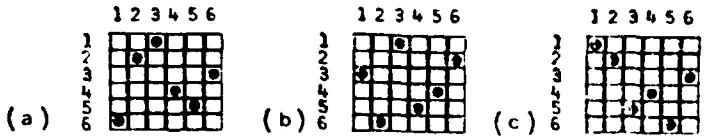

FIG. 1. Three examples of Costas Arrays of degree 6.

As a consequence of all these constructions, n×n Costas Arrays are now known to exist for the following positive values of n:

(i) $n = p - 1$, p prime,

(ii) $n = q - 2$, $q = p^k$ (any prime power),

(iii)* $n = q - 3$, $q = p^k$ (any prime power),

(iv)* $n = 2^k - 4$, $k \geq 3$.

Cases (iii) and (iv) depend on the conjecture that the field of q elements, $q > 2$, contains primitive roots α and β (not necessarily distinct) for which $\alpha + \beta = 1$. This conjecture has been verified for all prime powers q on the range $2 < q \leq 2^{11} = 2048$, and is discussed further in Section 3.

As an application of these results, algebraic constructions for n×n Costas Arrays are known for all $n \leq 130$ except for n = 19, 31, 32, 33, 27, 43, 48, 49, 53, 54, 55, 63, 67, 73, 74, 75, 83, 84, 85, 89, 90, 91, 92, 93, 97, 103, 109, 113, 114, 115, 116, 117, 120, 121, 127. However, the great majority of all integers n will not be included in these four classes of known constructions. It is hoped that this paper will inspire others to discover additional constructions for Costas Arrays. The major new constructions starts with Theorem 3 as follows:

Theorem 3. Let α and β be primitive elements in the field GF(q), for any $q > 2$. Then the $(q - 2)\times(q - 2)$ permutation matrix with $a_{ij} = 1$ iff $\alpha^i + \beta^j = 1$ is a Costas Array. (This reduces to Theorem 2 in the special case $\alpha = \beta$.)

References

1. J.P. Costas, "Medium constraints on sonar design and performance," *EASCON Convention Record* (1975), 68A-68L.

2. S.W. Golomb and H. Taylor, "Two-dimensional synchronization patterns for minimum ambiguity," *IEEE Trans. Inform. Theory*, IT-28, No. 4, (1982).

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-Solomon W. Golomb and Herbert Taylor, "Constructions and Properties of Costas Arrays", *Proceedings of the IEEE*, Vol. 72, No. 9, September 1984.

CONSTRUCTIONS AND PROPERTIES OF COSTAS ARRAYS

ABSTRACT

A Costas array is an n×n array of dots and blanks with exactly one dot in each row and column, and with distinct vector differences between all pairs of dots. As a frequency-hop pattern for radar or sonar, a Costas array has an optimum ambiguity function, since any translation of the array parallel to the coordinate axes produces at most one out-of-phase coincidence.

We conjecture that n×n Costas array exist for every positive integer n. Using various constructions due to L. Welch, A. Lempel, and the authors, Costas arrays are shown to exist when $n = p - 1$, $n = q - 2$, $n = q - 3$, and sometimes when $n = q - 4$ and $n = q - 5$, where p is a prime number, and q is any power of a prime number.

All known Costas array constructions are listed for 271 values of n up to 360. The first eight gaps in this table occur at n = 32, 33, 43, 48, 49, 53, 54, 63. (The examples for n = 19 and n = 31 were obtained by augmenting Welch's construction.)

Let C(n) denote the total number n×n Costas arrays. Costas calculated C(n) for $n \leq 12$. Recently, John Robbins found C(13) = 12828. We exhibit all the arrays for $n \leq 8$. From Welch's construction, $C(n) \geq 2n$ for infinitely many n.

Some Costas arrays can be sheared into "honeycomb arrays". All known honeycomb arrays are exhibited, corresponding to n = 1, 3, 7, 9, 15, 21, 27, 45.

Ten unsolved problems are listed.

COMPUTER

Two of three VAX-750's purchased on our DoD University Instrumentation Grant have been connected to the local computer net and are fully operational.

Our new dedicated computer system has definitely allowed us to carry out research which would have been prohibitively expensive if time were purchased for this effort. For example, three distinct kinds of searches, using sophisticated backtrack programs, have been carried out which have led to new results:

1. PPM sequence designs: Exhaustive search through 13x14 arrays took 228 hours of CPU time.

2. Costas Arrays: Exhaustive search through 13x13 arrays took 58 hours of CPU time.

3. Sonar Arrays: Exhaustive search through 11x17 arrays used 251 hours.

Extensions of the Costas array search are now being undertaken cooperatively with researchers at other organizations.

We hope to obtain an array processor for our VAX-750's in order to carry out research activities involving the coupling in phased-array antenna systems and spread spectrum receivers, and several aspects of sonar signal processing.

TRAVEL

During this reporting period, Dr. S.W. Golomb participated in the following meetings and workshops:

1. The Institute of Management Sciences (TIMS XXVI) International meeting, Copenhagen, Denmark, June 17-21, 1984. He delivered a presentation entitled "Information and Control in Management Systems," Session WB3 on June 20, 1984.

2. The Information Theory Workshop, Caesarea, Israel, July 1-5, 1984.

3. XXIst General Assembly of the International Scientific Radio Union (U.R.S.I.) in Florence, Italy, from August 28 to September 5, 1984, as an official representative of the United States National Academy of Sciences - National Research Council. *Partial travel support for this trip was provided by ONR with prior approval.*

Abstracts of Dr. Golomb's talks in Israel and Italy appear below.

Frequency Hop Patterns with Thumb-Tack Ambiguity Functions

M.J. Sites (1969) and J. Costas (1975) have posed the problem of finding n×n frequency hop patterns (n adjacent frequencies assigned to n consecutive time intervals in some permuted order) with the "thumb-tack property" that any non-zero shift of the pattern in time and/or frequency will result in at most one coincidence between occupied cells in the shifted and unshifted pattern. Systematic constructions for such hop patterns have been found by Welch, Lempel, Taylor, and Golomb, as described by Golomb and Taylor (1982) and Golomb (1984). In particular, n×n patterns can now be systematically constructed whenever n+1 is a prime, whenever n+2 is a prime or power of a prime, whenever n+4 is a power of two, and in many other cases as well. These patterns are directly applicable to signal design problems for frequency-hopped radar and sonar, and as two-dimensional synchronization patterns. They are also closely related to the patterns which arise in the synthetic aperture design problem for radio astronomy telescopes.

References

1. J. Costas (1975), "Medium Constraints on Sonar Design and Performance," in *EASCON Convention Record*, 1975, pp. 68A-68L.

2. S. Golomb (1984), "Algebraic Constructions for Costas Arrays," *Journal of Combinatorial Theory (A)*, vol. 37, no. 1, July 1984 (to appear).

3. S. Golomb and H. Taylor (1982), "Two-Dimensional Synchronization Patterns

for Minimum Ambiguity," *IEEE Trans. on Information Theory*, vol. IT-28, no. 4, July, 1982, pp. 600-604.

4. M.J. Sites (1969), "Coded Frequency Shift Keyed Sequences with Applications to Low Data Rate Communication and Radar," Technical Report 3606-5, *Radioscience Laboratory*, Stanford Electronics Laboratories, Stanford, California, September 1969. SU-SEL-69-0033.

Construction of Frequency Hop Patterns

A Sites-Costas (S-C) array is an n×n array of dots and blanks with exactly one dot in each row and column, and with distinct vector differences between all pairs of dots. As a frequency hop pattern for radar or sonar, an S-C array has an optimum ambiguity function, since any translation of the array parallel to the coordinate axes produces at most one out-of-phase coincidence.

We conjecture that n×n S-C arrays exist for every positive integer n. Using various constructions due to L. Welch, A. Lempel, H. Taylor, and the author, S-C arrays are shown to exist when $n=p-1$, $n=q-2$, $n=q-3$, and sometimes when $n=q-4$ and $n=q-5$, where p is a prime number, and q is any power of a prime number.

There are known S-C array constructions for 271 of the values of n up to 360. The first eight gaps occur at n=32, 33, 43, 48, 49, 53, 54, 63. (Examples for n=19 and n=31 were obtained by augmenting Welch's construction.)

Let C(n) denote the total number of n×n S-C arrays. Costas calculated C(n) for $n \leq 12$, with C(12)=7852. From Welch's construction, $C(n) \geq 2n$ for infinitely many n. Many unsolved problems regarding C(n) remain.

In 1966, Yates and Cooper proposed the problem of finding n simultaneous n×n permutation matrices, each regarded as a frequency hop pattern, such that the time cross-correlation between any pair of patterns never produces more than one coincidence for any time shift. There is an interesting relationship, involving the symmetries of a Latin Square, between the Yates-Cooper construction which solves this problem when n+1 is prime, and the Welch construction for Sites-Costas arrays.

SCIENTIFIC PERSONNEL SUPPORTED

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